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Comparison Subband Adaptive Polyphase Architecture with Mean Square Error

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Abstract- Subband adaptive polyphase convergence speed and complexity comparison have been recently apply for subband adaptive filter architecture. In this section, the convergence speed of all delayless subband adaptive filtering algorithms is compared. The convergence time can be measured by calculating a time constant based on the eigen values from the convergence analysis. The time constant τi is the time required for the k-th natural mode to reach 1/e of its value . In this comparison, the largest time constant corresponding to slowest mode, is compared for different delayless subband adaptive filters with number of subbands M = 64, 128, 128 , 256 and fullband filter length $L_f \leq 256$. This techniques have been recently used for subband adaptive filters, since some of the applications such as acoustic echo cancellation and wideband active noise control need adaptive filters. the delayless subband adaptive filter architecture open loop and closed loop convergence have been introduced. Delayless subband adaptive filtering is used in both open loop and closed loop configuration, where the subband filters are transformed to a fullband filter using a weight.

Keywords- **convergence, least mean square, intersymbole interference, lms, critically subband sampled.**

I. INTRODUCTION

Since μ is set such that $|1 - \mu\lambda k| < 1$, then the largest time constant τmax, which is related to the slowest mode and therefore important for the overall convergence behavior, is due to the eigen value for which the quantity $|1 - \mu\lambda i|$ is closest to unity. The maximum convergence speed is obtained when $\mu = 1/\lambda max$. The corresponding time constant for the fastest mode is then equal to zero. The minimum eigen value corresponds to slowest converging mode. It should be noted that the time constant unit is related to the update rate of the adaptive coefficients and must be multiplied with D to relate it to the full band sampling rate. The time constant τi is the time required for the k-th natural mode to reach 1/e of its value [1].

$$(1-\mu\lambda_i)^{\tau i} = 1/e \tag{1.1}$$

Taking the natural logarithm on both sides yield

$$\tau_i = \frac{-1}{\ln(1-\lambda\mu_i)} \tag{1.2}$$

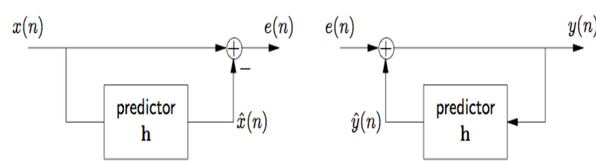

Fig 1. Subband adaptive open loop filtering

The largest time constants and time constant ratios are plotted [2]. As concluded earlier, the coefficient transform does not influence these results. The step size is set at, $\mu = 1/\lambda max$. It can be observed that for scenarios with a large number of fullband filter coefficients, the slowest mode of the LMS adaptive filter is slower than the slowest mode of the delayless subband adaptive filter. This is not necessarily the case with only few filter coefficients. It can also be seen that a distinct improvement is obtained [3].

When the number of subbands is increased towards M = 32. It should be noted that in the open loop case, the slowest mode does not necessarily influence the convergence in terms of the mean square error. It does influence the convergence of the subband filter coefficients.

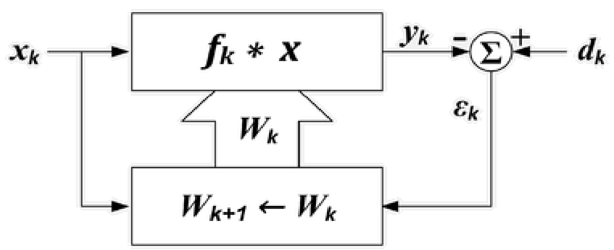

Fig 2. subband adaptive closed loop filtering

II. SUBBAND ADAPTIVE FILTERING WITH LEAST MEAN SQUARE

One of the most well-known control algorithm for adaptive filters is the Least Mean Square algorithm [1]. The LMS algorithm can in short be summarized using the equations

$y(n) = f^T(n)x(n)$ (2.1)

$e(n) = d(n) - y(n)$ (2.2)

$f(n + 1) = f(n) + \mu x^*(n)e(n)$ (2.3)

Where $f(n) = [f_0(n), \ldots, f_{Lf-1}(n)]^T$ is a vector with the filter coefficients at time instant n, and Lf denotes the number of filter coefficients. The vector

$x(n) = [x(n), \ldots, x(n-Lf+1)]^T$ is an input signal vector at time instant n, which holds Lf input samples starting with x(n) [4].

The LMS adaptive filter is an adaptive solution to the FIR Wiener filter design problem. The FIR Wiener filter is an optimal filter, which minimizes the Mean-Square Error

$J = E\{|e(n)|^2\}$ (2.4)

Where E{.} is the expectation operator. The Minimum Mean-Square Error and corresponding optimal coefficients found by taking the gradient with respect to the filter coefficients and setting it to zero [5].

$\nabla E\{|e(n)|^2\} = E\{e(n)\nabla e^*(n)\} = -2E\{e(n)x^*(n)\}$ (2.5)

Inserting eqations (2.1) and (2.2) and setting the gradient to zero yields the system of equations

$-2E\{d(n)x^*(n)\} + 2E\{x^*(n)xT(n)\}f = -2r_{dx} + 2R_{xx}f = 0$ (2.6)

Where matrix Rxx = $E\{x^*(n)x^T(n)\}$ is the input signal autocorrelation matrix, vector $r_{dx} = E\{d(n)x^*(n)\}$ is a cross-correlation vector. Solving Eq. (2.6) leads to the optimal Wiener filter

$f_{Wiener} = R^{-1}{}_{xx}r_{dx}$ (2.7)

The coefficients of the adaptive LMS algorithm are updated using an instantaneous estimate of the gradient.

III. SUBBAND ADAPTIVE CONVERGENCE IN THE MEAN SQUARE

In this section, the theoretic analysis of the convergence of the adaptive LMS filter in the mean is briefly described. It is shown that the convergence speed of the adaptive [6] filter is dependent on the properties of the input correlation matrix. Substituting Eqs. (2.1) and (2.2) into Eq. (2.3) gives

$f(n + 1) = f(n) + \mu d(n) - f^T(n)x(n)x^*(n)$ (2.8)

Taking the expected value,

$f(n+1) = f(n) + \mu E\ \{d(n)x^*(n)\} - \mu Ex^*(n)x^T(n)f(n)$ (2.9)

Under the assumption that the data x(n) and the LMS coefficient vector f (n) are statistically independent Eq. (2.9) can be rewritten as (2.10)

$f(n+1)= f(n) + \mu E\{d(n)x^*(n)\}-\mu Ex^*(n)x^T(n)\underline{f}(n)$

$= (I - \mu Rxx)f(n) + \mu rdx$ (2.11)

Replacing r_{dx} by $R_{xx}f_{Wiener}$ where $f_{Wiener}$is the optimal Wiener filter, yields

$f(n + 1) = (I - \mu_{Rxx})f(n) + \mu R_{xx}f_{Wiener}$ (2.12)

The coefficient error $\Delta f(n)$ is defined as

$\Delta f(n) = f(n) - f_{Wiener}$ (2.13)

Inserting Eq. (2.12) into Eq. (2.13) gives

$\Delta f(n) = (I-\mu R_{xx})f(n - 1) + \mu R_{xx}f_{Wiener} - f_{Wiener}$ (2.14)

$= (I - \mu R_{xx})(f(n - 1) - f_{Wiener})$ (2.15)

$= (I - \mu R_{xx})\Delta f(n - 1)$ (2.16)

Since the correlation matrix is hermitian, i.e. $R_{xx} = R^H{}_{xx}$ the matrix can be factorized using the eigen value decomposition $R_{xx} = V\Lambda V^H$ (the spectral theorem) with orthogonal eigenvector matrix V and diagonal matrix Λ

with real eigen values on the main diagonal. Using the eigen value decomposition and the fact that $VV^H = I$, yields

$$\Delta f(n) = (VV^H - \mu V\Lambda V^H)\Delta f(n-1) \quad (2.17)$$

$$= V(I - n\mu\Lambda)V^H \Delta f(n-1) \quad (2.18)$$

A modal coefficient error vector is introduced as

$\Delta f(n) = V^H \Delta f(n)$ and evolves as a function of time according to

$$\Delta f(n) = (I - \mu\Lambda)\Delta f(n-1) \quad (2.19)$$

With an initial vector $\Delta f(0)$ Eq. (2.19) can be rewritten as

$$\Delta f(n) = (I - \mu\Lambda) n \Delta f(0) \quad (2.20)$$

Since $(I - \mu\Lambda)$ is a diagonal matrix, the elements of $\Delta f(n)$ can be expressed as

$$\Delta f_i(n) = (1 - \mu\lambda_k) n \Delta f_i(0) \quad (2.21)$$

which are referred to as the natural modes of the adaptive filter. The time constant τi is the time required for the k-th mode to reach 1/e of its value

$$(1 - \mu\lambda i)^{\tau i} = 1/e \quad (2.22)$$

Taking logarithms yields

$$\tau_i = \frac{-1}{\ln(1 - \lambda\mu_i)} \quad (2.23)$$

In order for f (n) to converge to f_{Wiener}, $\Delta f(n)$ should converge to zero and therefore $\Delta f(n)$ should converge to zero. This will occur if and only if

$$|1 - \mu\lambda i| < 1, \forall i. \quad (2.24)$$

The decay for each mode is dependent on the magnitude of $|1 - \mu\lambda i|$ and is thus dependent on both μ and λi Therefore the step-size is restricted by,

$$0 < \mu < 2/\max\lambda i \quad (2.25)$$

IV. SIMULATED AND MEASURED RESULT

4.1 Open loop delayless subband filter result

The identification of a length *Np* = 64, 128, 128, 256 FIR system is considered. Experiments open loop delayless filter were performed with the fullband normalized LMS. The step-sizes were selected such that the best convergence rate μ= 0.1 presents the MSE evolutions. The new delayless subband structure presents a better convergence rate than the LMS algorithm, due to the power normalization of the step-sizes [7].

Table4.1

Number of subband M varying according to length

K	M	μ	N
64	8	0.1	500
128	8	0.1	500
128	16	0.1	500
256	16	0.1	500

Members of ensemble = K, Convergence factor = μ, Number of subband = M , Iteration = N. It converges to an MSE of the order of the stopband attenuation of the analysis filter which is around -12 db. Here small factor for the update equation for the signal energy in the subband .Coefficient vector of plant (denominator) A =1, coefficient vector of plant (numerator), B=[0.32,-0.3,0.5,0.1]

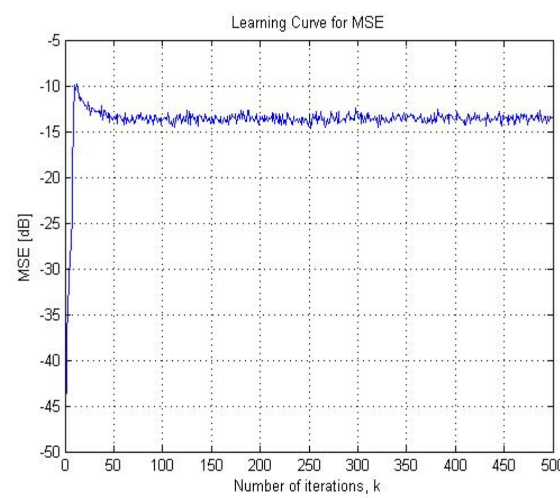

Fig.4.1.1: Simulation result of open loop system M=8 Np=64

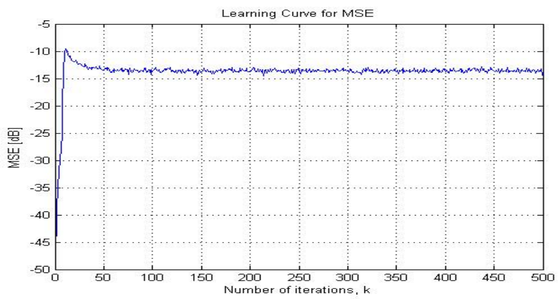

Fig.4.1.2:Simulation result of open loop system M=8 Np=128

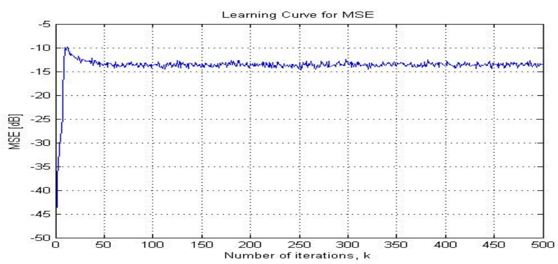

Fig.4.1.3 Simulation result of open loop system M=16 Np=128

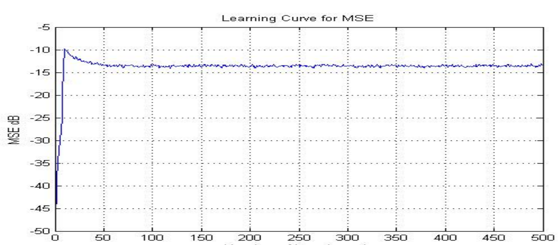

Fig.4.1.4: Simulation result of open loop system M=8 Np=256

4.2 Closed loop delayless subband adaptive result

The identification of a length *Np* = 64, 128, 128, 256 FIR system is considered. Experiments closed loop delayless filter were performed with the fullband normalized LMS. The step-sizes were selected such that the best convergence rate μ= 0.1 presents the MSE evolutions. The new delayless subband structure presents a better convergence rate than the LMS algorithm, due to the power normalization of the step-sizes. It converges to an MSE of the order of the stopband attenuation of the analysis filter which is around -11 db. Here small factor for the update equation for the signal energy in the subband .Coefficient vector of plant (denominator) A =1, coefficient vector of plant (numerator) B=[0.32,-0.3,0.5,0.2] [8].

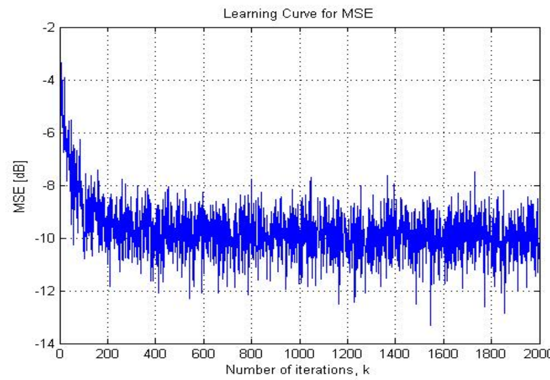

Fig.4.2.1 Simulation result of closed loop system M=8 Np=64

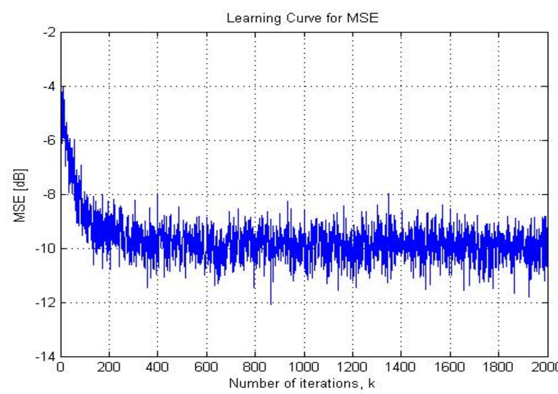

Fig.4.2.2 Simulation result of closed loop system M=8 Np=128

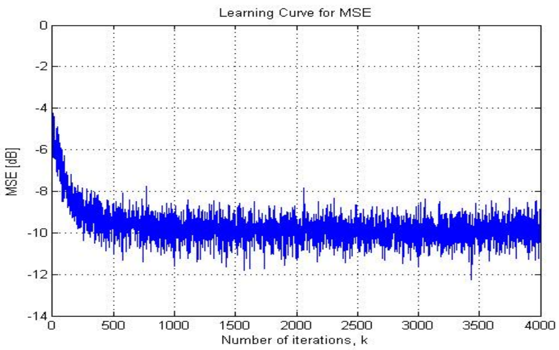

Fig.4.2.3: Simulation result of closed loop system M=16 Np=128

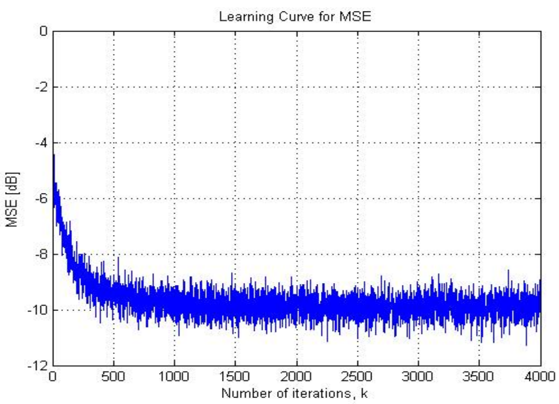

Fig.4.2.4 Simulation result of closed loop system M=16 Np=256

V. CONCLUSIONS & FUTURE WORK

The convergence rate behavior of the open loop and closed loop configurations of the delayless subband adaptive filters architecture is studied. It is shown that the subband to fullband transform greatly affects the performance in terms of the fullband mean square error for the open loop configuration and in terms of the convergence speed for the closed loop configuration. It is shown that based on the results for the closed loop case, a transform with optimal convergence performance can be derived. A novel delayless subband adaptive filter is presented, which employs polyphase adaptive filters. This convergence has been analyses and compared to the behavior of the fullband LMS algorithm through of computer MATLAB simulations. It can be observed that initially the oversampled subband structure presents better convergence rate. A closed loop structure with the following features has been proposed [9]:

1. Less MSE curve .
2. Better convergence .

Comparison of Measured MSE's Open Loop and Closed Loop

Table 5.1

Number of subband M varying according to length

Np	M	µ	Open loop MSE dB	Closed loop MSE dB
64	8	0.1	-14	-12
128	8	0.1	-13	-11
128	16	0.1	-12	-10
256	16	0.1	-12	-8

Members of ensemble =Np, Number of subband M, Convergence factor= µ

VI. REFERENCES

[1] Gilloire and M. Vetterli, "Adaptive Filtering in Subbands with Critical Sampling: Analysis, Experimentand Application to Acoustic Echo Cancellation," *IEEE Trans. Signal Processing,* pp. 1862- 1875, vol. 40, no. 8, Aug. 1992.

[2] S. S. Pradhan and V. U. Reddy, "A New Approach to Subband Adaptive Filtering,"*IEEE Transactions on Signal Processing*, vol. 47 no 3 pp. 655–664, March 1999.

[3] J. Huo, S. Nordholm, and Z. Zang, "New Weight Transform Schemes for DelaylessAdaptive Filtering," in *IEEE Global Telecommunications Conference*, 2001,pp. 197–201.

[4] M. R. Petraglia and S. K. Mitra, "Performance Analysis of Adaptive Filter Structures based on Subband Decomposition," in *Proc. IEEE Int. Symp. Circ. Syst.*, , Chicago, IL, May 1993.

[5] D. R. Morgan and J. C. Thi, “A Delayless Subband Adaptive Filter Architecture,”*IEEE Transactions on Signal Processing*, vol 43 no 8 pp. 1818–1830, August 1995.

[6] M. R. Petraglia, R. *G*. Alves and P. S. R. Diniz, “New Structures for Adaptive in Subbands with Critical Sampling,” *IEEE Trans. on Signal Processing,* vol. 48, No. 12, pp. 3316-3327, Dec. 2000.

[7] P. A. Naylor, O. Tanrikulu, and A. G. Constantinidis, “Subband Adaptive Filtering for Acoustic Echo Control using Allpass Polyphase IIR Filter Banks,” *IEEE Transactions on Speech and Audio Processing*, vol. 6, no. 2, pp. 143–155.

[8] N. Hirayama, H. Sakai, and S. Miyagi, “Delayless Subband Adaptive Filtering using the Hadamard Transform,” *IEEE Transactions on Signal Processing*, vol. 47, no. 6, pp. 1731–1734, June 1999.

[9] R. Merched, P. S. R. Diniz, and M. R. Petraglia, “A New Delayless Subband Adaptive Filter Structure,” *IEEE Trans. On .Signal Processing,* vol. 47. No. 6 1580-1591, Jun. 1999.